INFLUENCE OF AN ENTROPY LAYER ON NONSTATIONARY PERTURBATION PROPAGATION IN THE BOUNDARY LAYER

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An asymptotic theory of flows with free interaction is developed in [1, 2]. The simple form of the equations and boundary conditions and the general form of the similarity law afford the possibility of its application to a sufficiently broad class of flows. Numerical solutions are obtained for some of these flows, problem formulations are presented for others, equations and boundary conditions are written down, or reasoning is presented about the nature of the flow. Stationary hypersonic flows of a viscous gas with entropy layers are investigated in [3]. Equations subject to nonstationary processes in the boundary layer with self-induced pressure are investigated in [4].

In this paper the theory of flows with free interaction is applied to an investigation of nonstationary hypersonic viscous gas flows with entropy layers.

Let us consider the hypersonic viscous gas flow around a plate of finite length l parallel to the free stream ($M_\infty \gg 1$). The gas parameters in the unperturbed stationary state are marked with the subscript ∞. We assume that the Reynolds number $Re_0 = \rho_0 u_\infty l/\mu_0$ is large. Here ρ, u, μ are the density, tangential velocity component, and dynamic viscosity coefficient, respectively, and the subscript 0 marks values of the parameters evaluated at the free-stream stagnation temperature. We shall consider $M_\infty Re_0^{-1/2} \ll 1$. Let there be an entropy layer of thickness δ_{ent}, i.e., an inviscid flow domain in which the stagnation enthalpy can be taken equal to its value in the hypersonic stream, while the stagnation pressure and the density are considerably less than the corresponding quantities in the hypersonic stream, between the boundary layer and the hypersonic stream. Hence, the number $M_{ent} \sim O(1)$ and varies smoothly between a certain supersonic value on the upper boundary layer boundary and the hypersonic value $M_\infty \gg 1$.

Let t denote the time; x, y, Cartesian coordinates; u, v, velocity vector coordinates along these axes; ρ, density; and p, pressure.

In conformity with the general theory [1, 2] and its application to the weak hypersonic interaction regime for a temperature factor $O(1)$ [5], and according to [3] we will consider that four domains with substantially distinct properties will be formed during the free interaction of a nonstationary boundary layer with the external flow. The viscosity and heat-conduction effects are small in the upper domain I, and there are no vortices in the flow. The influence of dissipative factors can also be neglected in the middle domains II (outer part of the boundary layer) and IV (enthalpy layer domain), but the velocity field is vortical. The domain III consists of narrow near-wall viscous boundary-layer zones in which small pressure drops because of the high value of the pressure gradient cause a change in the velocity of the same order as the velocity itself. Viscosity plays a definite part in the formation of the flow in this domain. As regards the heat conduction, its role is secondary, at least for the temperature factor $\sim O(1)$, since gas compressibility is not manifest for law velocities. The outer part of the boundary layer (the domain II) does not exert any substantial influence on the flow in a first approximation for $g_w \sim 1$ (g_w is a temperature factor).

According to [3], we have the following estimates for the flow functions:

$$\begin{gathered}\Delta p/p \sim (M_\infty \varepsilon)^{1/2},\ \Delta x/l \sim (M_\infty \varepsilon)^{3/4},\\ \delta_3/l \sim \varepsilon (M_\infty \varepsilon)^{1/4},\ \varepsilon = \delta_0/l \sim Re_0^{-1/2},\\ \delta_4/l \sim \varepsilon/(M_\infty \varepsilon)^{1/4},\ u_3/u_\infty \sim (M_\infty \varepsilon)^{1/4}.\end{gathered} \tag{1}$$

Here and below the number of the domain to which the appropriate function is referred is

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marked by a subscript, for instance δ_3 is the thickness of domain III (δ_0 is the thickness of the unperturbed boundary layer).

The similarity parameter characterizing the role of the entropy layer in the interaction process can be written in the form $N = M_\infty\delta_4/(M_\infty\varepsilon)^{1/4}$. If we set $t = (l/u_\infty)[t_0 + (M_\infty\varepsilon)^{1/2}t_i]$, $i = 1, 2, 3, 4$, then it is essential that all the equations that describe processes in the different domains III should not contain derivatives with respect to the time. This means that the flows in domains I, IV, and II will behave inertly, succeeding in adjusting instantaneously to those perturbations that occur in the near-wall domain III. This result is obtained in [4], it is true, but under a certain other normalization for the time. The estimates calculated above permit introduction of asymptotic representations for the flow functions and formulation of the boundary-value problems by following the method in [1, 2].

Let us introduce the following variables

$$x = X\frac{\rho_w}{\mu_w a^2}\left(\frac{a\mu_w u_\infty^2}{\rho_w M_\infty^2}\frac{\rho_\infty}{\rho_w}\right)^{3/4}, \quad u = \left(\frac{a\mu_w u_\infty^2}{\rho_w M_\infty^2}\frac{\rho_\infty}{\rho_w}\right)^{1/4} U, \tag{2}$$

$$\Delta p = \rho_w\left(\frac{a\mu_w u_\infty^2}{\rho_w M_\infty^2}\frac{\rho_\infty}{\rho_w}\right)^{1/2} P,\ a = \left(\frac{\partial u}{\partial y}\right)_{0w},$$

$$\delta_3 = \frac{1}{a}\left(\frac{a\mu_w u_\infty^2}{\rho_w M_\infty^2}\frac{\rho_\infty}{\rho_w}\right)^{1/4}\Delta^*, \quad y = \frac{1}{a}\left(\frac{a\mu_w u_\infty^2}{M_\infty^2\rho_\infty}\frac{\rho_\infty}{\rho_w}\right)Y, \quad \Delta y_4 = \frac{lL_{\text{ent}}\rho_w}{\rho_\infty}\left(\frac{a\mu_w u_\infty^2}{\rho_w M_\infty^2}\frac{\rho_\infty}{\rho_w}\right)\Delta_4,$$

$$L_{\text{ent}} = \frac{1}{\gamma l}\int_0^\infty\left(\frac{1}{M_4^2} - 1\right)dy < 0, \quad t = \frac{\rho_w}{\mu_w a^2}\left(\frac{a\mu_w u_\infty^2}{\rho_w M_\infty^2}\frac{\rho_\infty}{\rho_w}\right)^{1/2}\tau.$$

The formulas (2) assure the passage to dimensionless variables simultaneously with the introduction of the asymptotic scales. For domain III we obtain

$$\begin{gathered}\partial U/\partial\tau + U\partial U/\partial X + V\partial U/\partial Y = -\partial P/\partial X + \partial^2 U/\partial Y^2,\\ \partial U/\partial X + \partial V/\partial Y = 0,\\ U = 0,\ V = 0 \text{ for } Y \to 0,\\ U = Y + A(\tau, X) \text{ for } Y \to \infty \text{ or } X \to -\infty.\end{gathered} \tag{3}$$

Merger with the solution for domains I, II, IV yields

$$P = \frac{d}{dX}(\Delta^* - N\Delta_4). \tag{4}$$

We write the variable part of the displacement thickness in the form

$$\Delta^* = \int_0^\infty\left(\frac{1}{U} - \frac{1}{\sqrt{2f}}\right)df = -A(\tau, X), \Delta_4 = P. \tag{5}$$

A derivation of this latter formula (5) is presented in [5]. The formula

$$N = \frac{al\,|L_{\text{ent}}|\,\rho_w}{p_\infty}\left(\frac{a\mu_w u_\infty^2}{\rho_w M_\infty^2}\frac{\rho_\infty}{\rho_w}\right)^{1/2} \tag{6}$$

can be written for the similarity parameter N. We shall seek the solution of the problem (3) and (4) in the form

$$U = Y - \alpha e^{\omega\tau + kX}\, df/dy,\ V = \alpha k e^{\omega\tau + kX},\ P = \alpha e^{\omega\tau + kX}, \tag{7}$$

where α is the perturbation amplitude. Linearization in the perturbation amplitude α reduces the problem (3) and (4) to the following form:

$$\begin{gathered}d^3f/dy^3 - (\omega + ky)df/dy + kf + k = 0,\\ f(0) = f'(0) = 0,\ f'(y) \to (1 + Nk)/k \text{ for } y \to \infty.\end{gathered} \tag{8}$$

We assume the constants ω and k in (7) to be complexes:

$$\omega = \omega_1 + i\omega_2,\ k = k_1 + ik_2.$$

To satisfy the limit conditions as $X \to -\infty$, it is sufficient to take $k_1 > 0$.

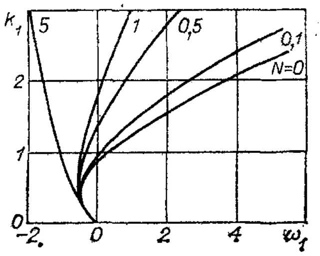

Fig. 1

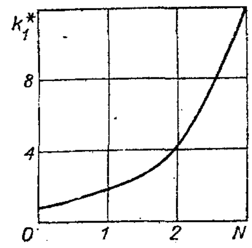

Fig. 2

The problem (8) is an eigenvalue problem. To solve the problem (8) we differentiate the first equation in (8) and perform a transformation $z = \omega/k^{2/3} + k^{1/3}y$ of the independent variable.

The solution satisfying the condition $|df/dz| < \infty$ is written in the form

$$f = -\left[\frac{d\,\mathrm{Ai}\left(\frac{\omega}{k^{2/3}}\right)}{dz}\right]^{-1} \int\limits_{\omega/k^{2/3}}^{\infty} \left[\int\limits_{\omega/k^{2/3}}^{z} \mathrm{Ai}(t)\,dt\right] dz,$$

where Ai(z) is the Airy function of a complex variable.

The last equation in (8) results in the dispersion relationship

$$-\frac{d\,\mathrm{Ai}\left(\frac{\omega}{k^{2/3}}\right)}{dz}(1+Nk) = k^{4/3} \int\limits_{\omega/k^{2/3}}^{\infty} \mathrm{Ai}(z)\,dz, \tag{9}$$

which interrelates the exponents k and ω. Values of these exponents that satisfy relationship (9) are intrinsic in the solution of the boundary-value problem (8). Problem (8) differs from the problem solved in [4] by the presence of the factor $(1 + Nk)$ that characterizes the role of the entropy layer, in the dispersion relationship (9). The influence of this factor on the value of the exponents ω and k are shown in Figs. 1 and 2. The dependence of k_1 on ω_1 is represented in Fig. 1 for $\omega_2 = k_2 = 0$. It should be noted that the curve in Fig. 1 is obtained by a direct conversion of the data in [4] if we set $\omega_1/k_1^{2/3} = \omega_{11}/k_{11}^{2/3}$, $k_1 = k_{11}/(1 + Nk_{11})^{3/4}$, where ω_{11}, k_{11} are the data in [4]. The curve for N = 0 corresponds to the results in [4]. Values of k_1^* are superposed in Fig. 2 for $k_2 = \omega_2 = 0$ and $\omega_1 = 0$ as a function of the parameter N. For the value $k = k_1^*$ the time dependence drops out in the linear solution of (8), and the gas flow in the boundary layer is stationary.

For $k > k_1^*$ the exponent is $\omega_1 > 0$, in this case the wave runs opposite to the main flow direction. For $k < k_1^*$ $\omega_1 < 0$. In this case the wave runs downstream. As N grows, k_1^* also grows. This latter is in agreement with known results [3] when greater and greater growth of the excess pressure along the X axis is necessary to assure upstream penetration of the perturbations as N grows.

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